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REFLECTION OF AN OBLIQUE SHOCK WAVE IN A REACTING GAS WITH A FINITE RELAXATION-ZONE LENGTH

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Reflection of an oblique shock wave in a reacting gas with a finite length of the chemical-reaction zone is studied. Shock polars for an arbitrary heat release behind the oblique shock wave are constructed. Transition criteria from regular to Mach reflection and back are obtained. It is shown that transition criteria are significantly changed if the reaction-zone length is taken into account.

Reflection of an oblique shock wave was considered in [1]. A review of the literature that deals with investigations of the transition from regular to Mach reflection is also given there, and disagreement between experimental and numerical data is noted. In the model of [1], the reaction-zone length is ignored, i.e., it is assumed that the reaction occurs on the shock-wave front in an infinitely thin zone. This assumption is valid in one-dimensional problems or problems with normal shock waves if the flow in the vicinity of the shock wave is not considered in detail. In the case of actual kinetics, the length of the chemical-reaction zone is finite. As is shown below, the neglect of the reaction-zone length leads to incorrect results, for example, in determining the transition criterion from regular to Mach reflection. Oblique detonation waves in an air–hydrogen mixture were numerically studied in [2–4]; the finite rate of chemical reactions for high values of the angle of flow deflection was taken into account in [3, 4].

In the present paper, we consider the model of a reacting polytropic gas with a characteristic ignition delay $\tau_i > 0$ and characteristic reaction time $\tau_r > 0$. Shock polars for an arbitrary degree of reaction completeness are constructed for analysis of reflection conditions of an oblique detonation wave. Based on the analysis of shock polars, conditions of transition from regular to irregular (Mach) reflection are obtained. It is shown that the transition conditions with a finite rate of chemical reactions taken into account are significantly different from the transition conditions in a gas with an instantaneous reaction.

Relations on the Oblique Shock Wave. We consider a plane steady flow of a reacting gas in the vicinity of the shock wave. A schematic of this flow is shown in Fig. 1.

The conservation laws on the oblique shock wave (Fig. 1) have the following form [5–7]:

$$\rho_2 u_{2n} = \rho_1 u_{1n}, \tag{1}$$

$$\rho_2 u_{2n}^2 + p_2 = \rho_1 u_{1n}^2 + p_1, \tag{2}$$

$$\frac{q_2^2}{2} + e_2 + \frac{p_2}{\rho_2} = \frac{q_1^2}{2} + e_1 + \frac{p_1}{\rho_1}, \tag{3}$$

$$u_{2\tau} = u_{1\tau}. \tag{4}$$

Here ρ_i is the density, u_{in} is the normal velocity, $u_{i\tau}$ is the tangential velocity, p_i is the pressure, e_i is the specific internal energy, and $q_i^2 = u_{in}^2 + u_{i\tau}^2$ is the velocity modulus squared; the subscript $i = 1$ and 2 refers to the conditions ahead of the shock wave and behind it, respectively.

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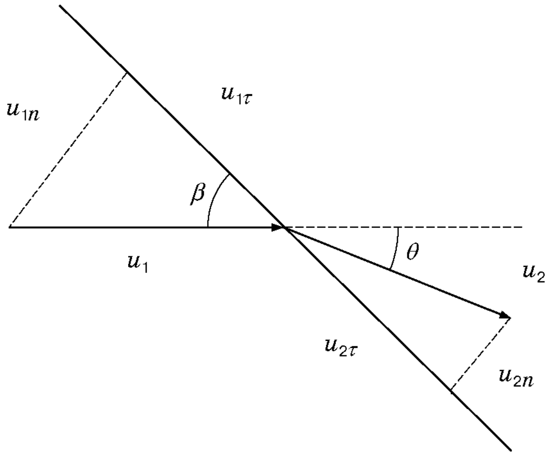

Fig. 1. Reflection of an oblique shock wave in a reacting gas.

The gases (initial and reacting) and the combustion products are assumed to be polytropic. The thermal effect of the reaction is assumed to be constant. Then, the internal energy of the gas is determined by the relations

$$e_1 = \frac{1}{\gamma_1 - 1}\frac{p_1}{\rho_1} + æ, \qquad e_2 = \frac{1}{\gamma_2 - 1}\frac{p_2}{\rho_2}, \tag{5}$$

where γ_1 and γ_2 ($1 < \gamma_2 \leqslant \gamma_1$) are the ratios of specific heats of the initial gas and reacted mixture, respectively, and $æ = \mathrm{const} \geqslant 0$ is the specific heat of reaction per unit mass of the gas.

Equation (2) may be written in the form

$$p_2 - p_1 = \rho_1 u_{1n}(u_{1n} - u_{2n}). \tag{6}$$

Equations (1)–(4) and (6) yield the Hugoniot adiabat equation

$$e_2 - e_1 = \frac{p_2 + p_1}{2}(V_1 - V_2), \tag{7}$$

where $V_i = 1/\rho_i$ ($i = 1$ and 2) is the specific volume of the gas.

The Hugoniot adiabat equation (7) is conveniently written in the form

$$V/V_1 = 1 - \Gamma(z), \tag{8}$$

where $\Gamma(z) = 2\{z + (\gamma_1 - \gamma_2)/[\gamma_1(\gamma_1 - 1)] - (\gamma_2 - 1)\bar{æ}_1/\gamma_1\}/[2\gamma_2/\gamma_1 + (\gamma_2 + 1)z]$, $\bar{æ}_1 = æ/(p_1V_1)$, $z = (J-1)/\gamma_1$ is the shock-wave amplitude, $J = p/p_1$ is the shock-wave strength, and V and p are the specific volume and the pressure behind the shock wave, respectively.

The equation for the Rayleigh straight line is

$$(1 - V_2/V_1)\mathrm{M}_1^2 \sin^2\beta = z, \tag{9}$$

where $\mathrm{M}_1 = u_1/c_1$ is the free-stream Mach number and $c_1^2 = \gamma_1 p_1 V_1$ is the velocity of sound. Eliminating the specific volume from Eqs. (8) and (9), we obtain

$$z = \Gamma(z)\mathrm{M}_1^2 \sin^2\beta. \tag{10}$$

The dependence between the angles θ and β is determined by the relation [6]

$$(\mathrm{M}_1^2 - z)\tan\theta = z\cot\beta. \tag{11}$$

Eliminating z from Eqs. (10) and (11), we obtain

$$\theta = \arctan\left[\cot\beta\,\frac{\gamma_1\mathrm{M}_1^2\sin^2\beta - \gamma_2 + \sqrt{\Psi}}{(\gamma_2 + \cos^2\beta)\gamma_1\mathrm{M}_1^2 + \gamma_2 - \sqrt{\Psi}}\right] \equiv \Theta(\beta; \mathrm{M}_1, \gamma_1, \gamma_2, \bar{æ}_1), \tag{12}$$

where $\Psi = (\gamma_1 \mathrm{M}_1^2 \sin^2\beta + 1)^2 - (\gamma_2^2 - 1)\{2[1/(\gamma_1 - 1) + \bar{æ}_1]\gamma_1 \mathrm{M}_1^2 \sin^2\beta - 1\}$. We denote the function inverse to (12) as

$$\beta = B(\theta; \mathrm{M}_1, \gamma_1, \gamma_2, \bar{æ}_1). \tag{13}$$

Using relations (1)–(10), we determine the following flow parameters behind the oblique detonation wave:

— ratio of pressures on the oblique shock wave

$$J \equiv \frac{p_2}{p_1} = \frac{1 + \bar{\Omega} + \sqrt{\Omega}}{\gamma_2 + 1}$$

$[\bar{\Omega} = \gamma_1 \mathrm{M}_1^2 \sin^2\beta$ and $\Omega = \bar{\Omega}^2 - 2\bar{\Omega}(\gamma_2^2 - 1)/(\gamma_1 - 1) + \gamma_2^2 - 2(\gamma_2^2 - 1)\bar{\Omega}\bar{æ}_1]$;

— ratio of specific volumes

$$\frac{V_2}{V_1} = 1 - 2\,\frac{J + (\gamma_1 - \gamma_2)/(\gamma_1 - 1) - (\gamma_2 - 1)\bar{æ}_1}{2\gamma_2 + (\gamma_2 + 1)J};$$

— Mach number behind the oblique shock wave

$$\mathrm{M}_2^2 = \frac{\gamma_1 \mathrm{M}_1^2}{\gamma_2 J}\,\frac{(1 - J)^2 \sin^2\beta + \cos^2\beta}{1 - 2[J + (\gamma_1 - \gamma_2)/(\gamma_1 - 1) - (\gamma_2 - 1)\bar{æ}_1]}.$$

Equation of the Shock Polar. It is convenient to analyze flows with oblique shock waves using shock polars [5, 7]. For the case of a reacting gas with heat release and changing ratio of specific heats behind the shock wave, shock polars have not been constructed previously.

Eliminating the angle β from (10) and (11), we obtain the shock-polar equation $J = J(\theta; \mathrm{M}_1, \gamma_1, \gamma_2, \bar{æ}_1)$ in the form of the implicit equation

$$\tan\theta = \frac{J - 1}{\gamma_1 \mathrm{M}_1^2 - (J - 1)}\sqrt{F - 1}, \tag{14}$$

where

$$F = \frac{2\gamma_1 \mathrm{M}_1^2}{(\gamma_1 - 1)(J - 1)}\,\frac{\gamma_1 - \gamma_2 + (\gamma_1 - 1)[J - 1 - (\gamma_2 - 1)\bar{æ}_1]}{(\gamma_2 + 1)(J - 1) + 2\gamma_2}.$$

Resolving the shock-polar equation (14) relative to J, we obtain the cubic equation

$$C_3 J^3 + C_2 J^2 + C_1 J + C_0 = 0, \tag{15}$$

where the coefficients $C_j = C_j(\theta; \mathrm{M}_1, \gamma_1, \gamma_2, \bar{æ}_1)$ ($j = 0$, 1, 2, and 3) have the following form:

$$C_0 = (\gamma_2 - 1)\{(\gamma_1 - 1)(\gamma_1 \mathrm{M}_1^2 + 1)^2 \tan^2\theta - 2\gamma_1 \mathrm{M}_1^2[(\gamma_1 - 1)\bar{æ}_1 + 1] + (\gamma_1 - 1)\},$$

$$C_1 = (\gamma_1 - 1)\{2\gamma_1 \mathrm{M}_1^2(\gamma_2 - 1)\bar{æ}_1 + (\gamma_1 \mathrm{M}_1^2 + 1)[\gamma_1 \mathrm{M}_1^2(\gamma_2 + 1) - (\gamma_2 - 3)]\tan^2\theta\}$$

$$+ 2\gamma_1 \mathrm{M}_1^2(\gamma_1 + \gamma_2 + 2) - (\gamma_1 - 1)(\gamma_2 - 3),$$

$$C_2 = -(\gamma_1 - 1)\{[2\gamma_1(\gamma_2 + 1)\mathrm{M}_1^2 + \gamma_2 + 3]\tan^2\theta + 2\gamma_1 \mathrm{M}_1^2 + \gamma_2 + 3\},$$

$$C_3 = (\gamma_1 - 1)(\gamma_2 + 1)(\tan^2\theta + 1).$$

From Eq. (15), we determine the inverse two-valued function $\theta = \bar{\Theta}(J; \mathrm{M}_1, \gamma_1, \gamma_2, \bar{æ}_1)$.

Equations (14) or (15) determine implicitly the shock polar $J = J(\theta; \mathrm{M}_1, \gamma_1, \gamma_2, \bar{æ}_1)$ in the plane (θ, J). As in [7], in addition to the shock-wave strength J, we consider the quantity

$$\Lambda = \ln J. \tag{16}$$

The shock polars (16) for inert and reacting gases are plotted in Fig. 2.

The shock polar $J = J(\theta; \mathrm{M}_1, \gamma_1, \gamma_2, \bar{æ}_1)$ (Fig. 2) has four special points. The limiting angle θ_l of flow deflection in the oblique shock wave and its strength J_l are determined from the relation $dJ(\theta_l; \mathrm{M}_1, \gamma_1, \gamma_2, \bar{æ}_1)/dJ = 0$. The point l on the shock polar separates the shocks into two families [7]:

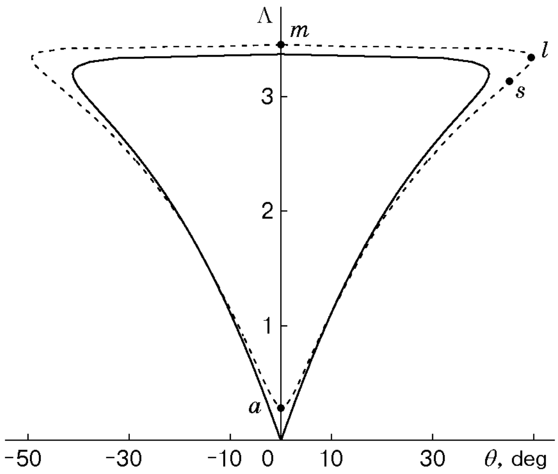

Fig. 2. Shock polars of inert gas [$\Lambda = \Lambda(\theta; \mathrm{M}_1, \gamma_1, \gamma_1, 0)$, solid curve] and reacting gas [$\Lambda = \Lambda(\theta; \mathrm{M}_1, \gamma_1, \gamma_2, \bar{æ}_1)$, dashed curve] for $\gamma_1 = 1.4$, $\gamma_2 = 1.2$, $\bar{æ}_1 = 4$, and $\mathrm{M}_1 = 5$: a is the beginning of the shock polar, m is the normal shock wave, l is the limiting angle of flow deflection behind the oblique shock wave, and s is the oblique shock wave with a sonic gas flow behind it.

a weak family ($J < J_l$) corresponding to the shock wave attached to the wedge and a strong family ($J > J_l$) corresponding to the flow around the wedge with a detached shock wave.

The point s on the shock polar corresponds to a shock wave with a sonic velocity behind it. For a shock-wave strength $J < J_s$, the flow behind the shock wave is supersonic; for $J > J_s$, the flow is subsonic. The coordinates of the point s are found from the condition $\mathrm{M}_2(\theta_s; \mathrm{M}_1, \gamma_1, \gamma_2, \bar{æ}_1) = 1$. In this case, we have

$$J_s = \frac{1}{2\gamma_2}\Big[\gamma_1 \mathrm{M}_1^2 - 2(\gamma_2 - 1)\bar{æ}_1 + \frac{2\gamma_1 - (\gamma_1 + 1)\gamma_2}{\gamma_1 - 1} + \sqrt{\Delta_s}\Big],$$

where

$$\Delta_s = \gamma_1^2 \mathrm{M}_1^4 + 2\,\frac{(\gamma_1 - 3)\gamma_2^2 - (\gamma_1 - 1)\gamma_2 + 2\gamma_1}{(\gamma_1 - 1)(\gamma_2 + 1)}\,\gamma_1 \mathrm{M}_1^2$$
$$+ \frac{(\gamma_1 + 1)^2\gamma_2^3 + (5\gamma_1^2 - 10\gamma_1 + 1)\gamma_2^2 - 4\gamma_1(2\gamma_1 - 1)\gamma_2 + 4\gamma_1^2}{(\gamma_1 - 1)^2(\gamma_2 + 1)}$$
$$- 4(\gamma_2 - 1)\Big[\gamma_1 \mathrm{M}_1^2 - (\gamma_2 - 1)\bar{æ}_1 - \frac{(\gamma_1 + 1)\gamma_2^2 + (\gamma_1 - 1)\gamma_2 - 2\gamma_1}{(\gamma_1 - 1)(\gamma_2 + 1)}\Big]\bar{æ}_1.$$

The strength of the normal shock wave J_m is determined by the expression $J_m = J(\pi/2; \mathrm{M}_1, \gamma_1, \gamma_2, \bar{æ}_1)$.

For a reacting gas ($\bar{æ}_1 > 0$ or $\gamma_1 \neq \gamma_2$), the shock polar begins at the point $J_0 > 1$ (Fig. 2). In this case, we have $J_0 = (\gamma_1 \mathrm{M}_1^2 + 1 - \sqrt{\Delta_0}\,)/(\gamma_2 + 1)$, where $\Delta_0 = \gamma_1^2 \mathrm{M}_1^4 - 2\gamma_1 \mathrm{M}_1^2[(\gamma_2^2 - \gamma_1)/(\gamma_1 - 1) + (\gamma_2^2 - 1)\bar{æ}_1] + \gamma_2^2$. Note that the shock polar in an inert gas ($\bar{æ}_1 = 0$ and $\gamma_1 = \gamma_2$) begins at the point $J_0 = 1$ (or $\Lambda_0 = 0$).

Reflection of an Oblique Shock Wave Without the Reaction Zone. Reflection of an oblique shock wave without the reaction zone is shown in Fig. 3. The gas ignites at the point F_0 downstream of the wedge apex (point A). The distance between the point F_0 and the wedge apex is equal to the length of the ignition-induction region. The ignition front F_0R_0 generates a compression wave $F_0H_0R_0$. In this wave, the ignition-induction time decreases. The compression wave interacts with the incident shock wave, which leads to wave curvature (sector H_0R_0). The strength of the shock wave H_0R_0 increases, and the ignition-induction time decreases. The structure of the curved shock wave with the compression wave is called the λ-structure [3, 4]. The angle of inclination of the incident shock wave at the wedge apex is β_0, and at the reflection point it is β_1. The relationship between the angles β_1 and β_0 is found from the relation $\Theta(\beta_1; \mathrm{M}_1, \gamma_1, \gamma_2, \bar{æ}_1) = \Theta(\beta_0; \mathrm{M}_1, \gamma_1, \gamma_1, 0)$ (the condition of equality of the angles of flow inclination behind the shock waves AH_0 and R_0T), which yields the dependence

$$\beta_1 = \bar{B}(\beta_0; \mathrm{M}_1, \gamma_1, \gamma_2, \bar{æ}_1). \qquad (17)$$

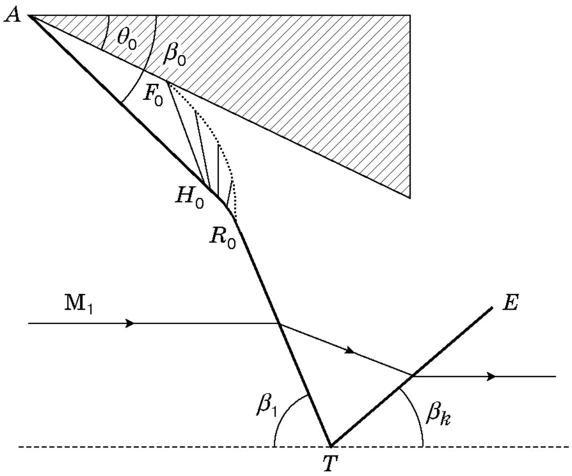

Fig. 3. Reflection of an oblique shock wave without the reaction zone.

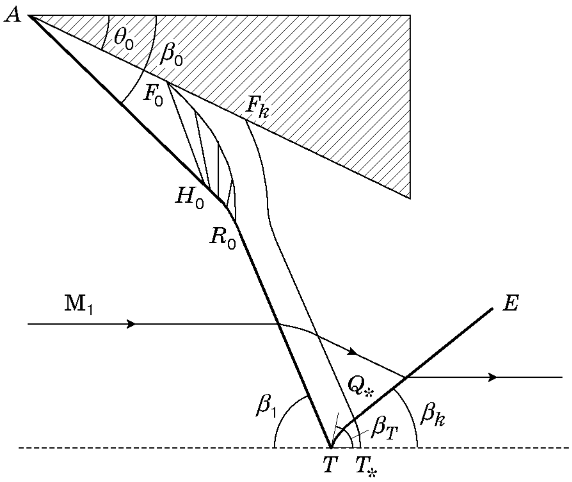

Fig. 4. Reflection of an oblique shock wave with the reaction zone ($F_0H_0R_0TT_*F_k$ is the combustion region).

We denote the inverse function as $\beta_0 = \tilde{B}(\beta_1; \mathrm{M}_1, \gamma_1, \gamma_2, \bar{æ}_1)$. The shock wave H_0T is reflected at the point T by an oblique shock TE (Fig. 3). The reaction occurs on the shock wave H_0T but is absent on the reflected shock TE. Hence, TE is an oblique shock wave in an inert gas with the ratio of specific heats γ_2. The angle of inclination of the reflected shock wave to the plane of symmetry β_k is determined by the relation

$$\beta_k = B(\theta_0; \mathrm{M}_2, \gamma_1, \gamma_2, \bar{æ}_2) - \theta_0, \tag{18}$$

where $\bar{æ}_2 = \bar{æ}_1 p_1 V_1/(p_2 V_2)$, M_2 is the Mach number, and p_2 and V_2 are the pressure and specific volume behind the incident shock wave after the reaction is completed.

Reflection of an Oblique Shock Wave with the Reaction Zone. Reflection of an oblique shock wave with the reaction zone is shown in Fig. 4 ($F_0H_0R_0TT_*F_k$ is the combustion region). The flow structure near the wedge surface is the λ-structure [3, 4] described above. The combustion-zone width depends on particular kinetics of the chemical reaction and is not important for further consideration. The angles β_1 and β_0 are related by Eq. (17).

Reflection of the incident shock wave with taking into account the reaction zone has a more complicated structure than without it. The shock-wave reflection occurs in the inert gas at the point T (reaction has not yet begun) with an angle of inclination β_T; the shock wave curves in the combustion zone (sector TQ_*); and after completion of the reaction, the angle of inclination becomes β_k [it is determined by Eq. (18)].

The angle of inclination of the reflected shock wave at the point T is $\beta_T = B(\bar{\theta}_0; \mathrm{M}_2, \gamma_1, \gamma_1, 0) - \theta_0$, where $\bar{\theta}_0 = \Theta(\beta_1; \mathrm{M}_1, \gamma_1, \gamma_1, 0)$ is the angle of inclination of the incoming flow for an inert gas, which corresponds to an oblique shock wave with an angle of inclination β_1. Note that the inequality $\beta_T > \beta_k$ is always satisfied. In the absence of the chemical reaction, we have $\beta_T = \beta_k$.

Criteria of Transition from Regular to Mach Reflection in an Inert Gas. The transition from regular to Mach reflection is determined by the criterion β_d: if the angle of inclination of the incident shock wave is $\beta > \beta_d$, regular reflection is impossible [5, 7–9]. The criterion β_d is called the two-shock theory criterion [8] or the detachment criterion [9]. The angle β_d is found from the following equation [5]:

$$\Big[\frac{(\gamma+1)\mathrm{M}_1^2}{2(\mathrm{M}_1^2\sin^2\beta_d - 1)} - 1\Big]\tan\beta_d = \Big[\frac{(\gamma+1)\mathrm{M}_{2d}^2}{2(\mathrm{M}_{2d}^2\sin^2\beta_{2d} - 1)} - 1\Big]\tan\beta_{2d}, \qquad (19)$$

where

$$\mathrm{M}_{2d}^2 = \frac{2 + (\gamma-1)\mathrm{M}_1^2}{2\gamma\mathrm{M}_1^2\sin^2\beta_d - (\gamma-1)} + \frac{2\mathrm{M}_1^2\cos^2\beta_d}{(\gamma-1)\mathrm{M}_1^2\sin^2\beta_d + 2},$$

$$\sin^2\beta_{2d} = \frac{1}{\gamma\mathrm{M}_{2d}^2}\left[\frac{\gamma+1}{4}\mathrm{M}_{2d}^2 - 1 + \sqrt{(\gamma+1)\Big(1 + \frac{\gamma-1}{2}\mathrm{M}_{2d}^2 + \frac{\gamma+1}{16}\mathrm{M}_{2d}^4\Big)}\right].$$

The transition condition from Mach to regular reflection in an inert gas is determined by the criterion β_N: if the angle of inclination of the incident shock wave is $\beta < \beta_N$, Mach reflection is impossible [5, 7–9]. The criterion β_N is called the von Neumann criterion [5], the three-shock theory criterion [8], or the mechanical-equilibrium criterion [9]. The angle β_N is found from the following equation [5]:

$$\cot^4\beta_N - \frac{\gamma\mu(\xi+\mu) + (1-\xi)^2}{(\xi+\mu)(1+\xi\mu)}\cot^2\beta_N - \frac{\gamma(\xi+\mu)}{(1+\xi\mu)^2} = 0. \qquad (20)$$

Here $\mu = (\gamma-1)/(\gamma+1)$ and $\xi = 1/[(\mu+1)\mathrm{M}_1^2\sin^2\beta_N - \mu]$.

Transition Criteria from Regular to Mach Reflection in a Reacting Gas. For a reacting gas with a reaction zone (see Fig. 4), the transition criteria should be determined separately for the point T and the shock wave Q_*E. Reflection in the inert gas occurs at the point T (we have $\gamma_1 = \gamma_2$ and $\bar{æ}_1 = 0$ in front of the shock wave and behind it). The critical angle β_1 is determined by the criterion β_d (19). Based on the angle β_1, the critical angle β_0 is determined. Thus, in the case of reflection at the point T, the critical angle β_0 is determined by the relations

$$\beta_1 = \beta_d^0(\mathrm{M}_1; \gamma_1), \qquad \beta_0 = \bar{B}(\beta_1; \mathrm{M}_1, \gamma_1, \gamma_2, \bar{æ}_1), \qquad (21)$$

where the function $\beta_d^0(\mathrm{M}_1; \gamma_1)$ is found from (19). Equations (21) determine the critical angle

$$\beta_0 = \beta_d(\mathrm{M}_1; \gamma_1, \gamma_2, \bar{æ}_1). \qquad (22)$$

For $\beta_0 > \beta_d(\mathrm{M}_1)$, regular reflection at the point T is impossible. The function $\beta_d(\mathrm{M}_1)$ is plotted in Fig. 5.

For the reflected wave Q_*E, the critical angle of regular reflection is determined by the following conditions:

1) the reflected flow is turned by an angle $\theta_0 = |\bar{\Theta}(J_3; \mathrm{M}_2, \gamma_2, \gamma_2, 0)|$, where $J_3 = J(\theta_0; \mathrm{M}_2, \gamma_2, \gamma_2, 0)$ is the reflected shock strength;

2) the flow is turned by a limiting possible angle on the reflected shock wave Q_*E, i.e., $J_3 = J_l(\mathrm{M}_2, \gamma_2, \gamma_2, 0)$.

Then, the critical angle $\beta_0 = B(\theta_0; \mathrm{M}_1, \gamma_1, \gamma_2, \bar{æ}_1)$ is found from (13). Finally, we obtain the transition criterion to Mach reflection, the reaction zone being ignored, in the form $\beta_0 = \tilde{\beta}_d(\mathrm{M}_1; \gamma_1, \gamma_2, \bar{æ}_1)$. For $\beta_0 > \tilde{\beta}_d(\mathrm{M}_1)$, regular reflection for the shock wave Q_*E is impossible. The function $\tilde{\beta}_d(\mathrm{M}_1)$ is plotted in Fig. 5. It is seen that $\beta_d(\mathrm{M}_1) < \tilde{\beta}_d(\mathrm{M}_1)$ for all free-stream Mach numbers M_1.

The transition from regular to Mach reflection occurs at $\beta_0 = \beta_d$, i.e., taking into account the reaction zone decreases the critical angle of transition. For angles $\beta_d < \beta_0 < \tilde{\beta}_d$, regular reflection in the vicinity of the point T is impossible.

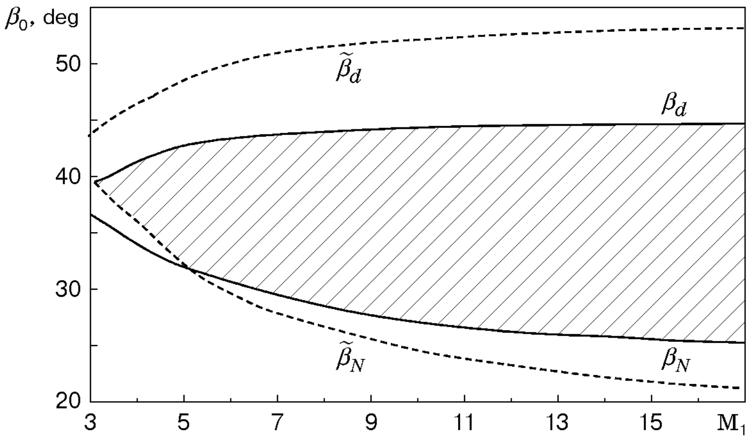

Fig. 5. Critical angles β_0 in a reacting gas ($\gamma_1 = 1.4$, $\gamma_2 = 1.2$, and $\bar{æ}_1 = 4$): $\beta_N(\mathrm{M}_1)$ and $\beta_d(\mathrm{M}_1)$ refer to the case with the reaction zone and $\tilde{\beta}_N(\mathrm{M}_1)$ and $\tilde{\beta}_d(\mathrm{M}_1)$ refer to the case without the reaction zone; the shaded region is the dual solution domain in a reacting gas, the upper boundary of the shaded region is the critical angle $\beta_d(\mathrm{M}_1)$ of transition from regular to Mach reflection, and the lower boundary is the critical angle $\bar{\beta}_N(\mathrm{M}_1) = \max(\beta_N(\mathrm{M}_1), \tilde{\beta}_N(\mathrm{M}_1))$ of transition from Mach to regular reflection.

The transition criterion from Mach to regular reflection [10, 11] is determined in a similar manner, i.e., separately for the reacting gas (for the “frozen” chemical reaction, we have $\gamma_1 = \gamma_2$ and $\bar{æ}_1 = 0$) and for the completely reacted gas.

For the “frozen” shock wave, we determine the critical angle β_1 from formula (20) and then the critical angle β_0. Thus, for Mach reflection of a “frozen” shock wave, the critical angle β_0 is determined by the relations

$$\beta_1 = \beta_N^0(\mathrm{M}_1; \gamma_1), \qquad \beta_0 = \bar{B}(\beta_1; \mathrm{M}_1, \gamma_1, \gamma_2, \bar{æ}_1), \tag{23}$$

where the function $\beta_N^0(\mathrm{M}_1; \gamma_1)$ is determined from (20). Equations (23) determine the critical angle $\beta_0 = \beta_N(\mathrm{M}_1; \gamma_1, \gamma_2, \bar{æ}_1)$. For $\beta_0 < \beta_N(\mathrm{M}_1)$, Mach reflection of a “frozen” shock wave is impossible. The function $\beta_N(\mathrm{M}_1)$ is plotted in Fig. 5.

For a completely reacted gas, the critical angle of Mach reflection is determined by the following conditions:

1) the reflected flow is turned by an angle $\theta_0 = |\bar{\Theta}(J_3; \mathrm{M}_2, \gamma_2, \gamma_2, 0)|$, where $J_3 = J(\theta_0; \mathrm{M}_2, \gamma_2, \gamma_2, 0)$ is the reflected shock strength;

2) the pressure behind the normal shock (Mach stem J_m) equals the pressure behind the system of two shocks (incident J_1 and reflected J_3 shock waves): $J_1(\mathrm{M}_1; \gamma_1, \gamma_2, \bar{æ}_1) J_3(\mathrm{M}_2; \gamma_2, \gamma_2, 0) = J_m(\mathrm{M}_1; \gamma_1, \gamma_2, \bar{æ}_1)$.

Then, the critical angle $\beta_0 = B(\theta_0; \mathrm{M}_1, \gamma_1, \gamma_2, \bar{æ}_1)$ is found from (13). Finally, we obtain the transition criterion to regular reflection, the reaction zone being ignored, in the form $\beta_0 = \tilde{\beta}_N(\mathrm{M}_1; \gamma_1, \gamma_2, \bar{æ}_1)$. For $\beta_0 < \tilde{\beta}_N(\mathrm{M}_1)$, Mach reflection for the reacted gas is impossible. The function $\tilde{\beta}_N(\mathrm{M}_1)$ is plotted in Fig. 5.

It follows from Fig. 5 that we have $\beta_N(\mathrm{M}_1) > \tilde{\beta}_N(\mathrm{M}_1)$ for $\mathrm{M}_1 \gtrsim 5$ and $\beta_N(\mathrm{M}_1) < \tilde{\beta}_N(\mathrm{M}_1)$ for $\mathrm{M}_1 \lesssim 5$. The curves $\beta_d(\mathrm{M}_1)$ and $\tilde{\beta}_N(\mathrm{M}_1)$ intersect at $\mathrm{M}_1 \approx 3$. The transition criterion from Mach to regular reflection is the maximum angle among β_N and $\tilde{\beta}_N$:

$$\bar{\beta}_N(\mathrm{M}_1) = \max(\beta_N(\mathrm{M}_1), \tilde{\beta}_N(\mathrm{M}_1)). \tag{24}$$

The range of angles from $\beta_d(\mathrm{M}_1)$ to $\bar{\beta}_N(\mathrm{M}_1)$ is the dual solution domain, where both regular and Mach reflection is possible.

Conclusions. The solution for regular reflection of an oblique shock wave in a reacting gas with taking into account the chemical-reaction zone and the shock polars [(14) or (15)] for shock waves with chemical reactions are constructed. Critical angles of transition from regular to Mach reflection $\beta_d(\mathrm{M}_1)$ [formula (22)] and from Mach to regular reflection $\bar{\beta}_N(\mathrm{M}_1)$ [formula (24)] for a reacting gas are derived. It is shown that taking into account the finite length of the chemical-reaction zone leads to a significant reduction of the dual solution domain (shaded region in Fig. 5).

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